Stochastic Models of the Stele

Model Setting and Simulation Algorithm

The longitudinal cross section of the stele is approximated as a 1-dimensional continuum interval [0, L] ∈ R, as shown in the figure below.

Initially, the 1-dimensional cell stoles are of roughly equal size, up to a small random perturbation. The locations of their distal (from the QC) ends are recorded as the points

\[ x_j = (x_j, v_j) \]

in [0, L], so \( x_j \) is the distal end of the j-th cell. The simulation proceeds as follows. The points (1) then move according to a velocity profile \( v(x) \), which satisfies

\[ v(x) = 0 \]

and generally alters the lengths of the cells. At every instant when a cell’s size exceeds a threshold length, \( s_j - s_{j-1} \geq \) threshold length \( d_l \),

(3)

this cell divides (in this simulation, exactly in half, but this need not be the case generally). The locations of the new cells’ distal ends are recorded.

The Choice of a Velocity Profile \( v(x) \)

We considered a priori the following three qualitatively different choices of \( v(x) \) (here denotes a constant):

\[ v(x) = \begin{cases} v_0 & \text{if } x < x^0 \cr v_1 & \text{if } x > x^0 \end{cases} \]

\[ v(x) = v_0 + (v_1 - v_0) \frac{x - x^0}{d_l} \]

(4)

in the framework of the above simulation. The first of these choices is unsuitable as it does not alter cell length, hence fails to capture growth. The third choice potentially leads to discontinuous motion, contradicting intuition and recorded as the points

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Experimental Methods and Analysis

In vivo time lapse recording of cell division in roots was performed using an automatic tracking method [3]. The experiment was run for approximately 15 hours. The obtained sequence of images was analyzed for the locations of the mitoses taking place in the observed cross section of the stile.

Simulations and Experimental Results

The following Figure shows experimental data (part A) juxtaposed with simulation results (parts B-D).

1-Dimensional Steady Motion Models of the Root

A Brief on 1-Dimensional Continuum Mechanics

Consider a rod of length \( l \) and cross section \( A \). In order to stretch or compress it lengthwise to a new length \( l' \geq l \), we achieve the fractional change in length

\[ \frac{\Delta l}{l} = F = \frac{\Delta l}{l} \]

(5)

(negative in the case of compression), one must apply a certain force \( F \), which depends on the required fractional stretch \( \frac{\Delta l}{l} = F \).

The dimensionless quantity \( \frac{\Delta l}{l} \) is called, strain. Larger strain requires larger force. Also, the thicker the rod, the larger the force required to achieve a strain \( \frac{\Delta l}{l} \).

Thus, \( F \) is proportional to \( \frac{\Delta l}{l} \). For many materials, these linear proportions are

\[ F = \frac{\Delta l}{l} \]

where the quantity \( \frac{\Delta l}{l} \) measures the stiffness of the material.

Rewrite (5) in the form

\[ \frac{\Delta l}{l} = \frac{1}{E} \]

(6)

The left-hand side is called stress. Equation (7) describes the rod’s stress response to a given strain \( \frac{\Delta l}{l} \). The intensity of this response is reflected by the constant \( E \), called Young’s modulus [2], which characterizes the stiffness of the material.

Model Setting

The longitudinal cross section of the outer cell layer is regarded as a 1-dimensional continuum. It is parameterized by a variable \( x \), which varies from \( 0 \) to \( l \) on the RAM (sample point on the RAM surface).

Equations

We consider the motion of this tissue fragment over a period of time comparable with one cell cycle. As a consequence of this, a point originally in position \( x + n(x) \) is displaced to a new location \( x + n(x) + \Delta x \), where \( n(x) + \Delta x \) is not necessarily equal to \( n(x) + \Delta x \), hence \( \Delta x \) can be arbitrarily small.

\[ \frac{\Delta x}{\Delta t} = v(x) \]

(9)

where \( v(x) \) is the velocity at position \( x \). This equation is an ordinary differential equation (ODE).

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Equations (9) can be rewritten as

\[ \frac{\Delta x}{\Delta t} = \left( \frac{\Delta x}{\Delta t} \right) \cdot \left( \frac{\Delta x}{\Delta t} \right) = v(x) \]

(10)

which predicts a rapid increase in stiffness towards the root tip (\( v = 0 \)). This result agrees with the intuitive hypothesis that near the root tip, longitudinal strain is necessarily accompanied by tangential strain, which is hindered by the transversely oriented cellulose fibrils. The result also correlates with the expansion concentation found to decrease towards the root tip in certain plants.