A ONE-DIMENSIONAL MODEL FOR THE REGULATION OF THE SIZE OF THE RENEWABLE ZONE IN BIOLOGICAL TISSUE

Nikolaev S.V.[*1], Fadeev S.I.[2], Kogai V.V.[2], Mjolsness E.[3], Kolchanov N.A.[1]

[1] Institute of Cytology and Genetics, SB RAS, Novosibirsk, 630090, Russia; [2] Institute of Mathematics, SB RAS, Novosibirsk, 630090, Russia; [3] School of Information and Computer Science and Institute for Genomics and Bioinformatics, University of California, Irvine, CA, USA
[*] Corresponding author: e-mail: nikolaev@bionet.nsc.ru

Key words: shoot apical meristem, SAM, tissue, morphogenesis, mathematical model

SUMMARY

Motivation: Spatial structure of biological tissues develops during growth and morphogenesis of an organism. In certain cases such a special structure remains invariant during the all organism's life despite constant renewal of the cells in this tissue. The question of interest is: what scheme underlies this phenomenon?

Results: A simple mathematical model for the regulation of shoot apical meristem compartments sizes was developed. A mathematical analysis had revealed numerous of solutions, some of which were biologically plausible and interesting.

INTRODUCTION

Differentiated cells make up the bulk of the cells of the adult organism. Each differentiated cell does not divide, as a rule, being specialized to perform a particular function in a particular organ tissue. However, in the adult organism, there exist cells that are undifferentiated, although perhaps "predetermined". This means that their fate is, to a certain extent, predestined in the sense that they can become cells of a particular type or of a restricted set of types There are stem cells that continue to divide at a definite rate. The stem cells are important for the life of the adult. In the animal tissue, they serve as sources of continuously renewable tissue (the skin, for example) and certain plants, the stem cells of the shoot meristem (the growing tip of the shoot) provides the growth of the plant through its entire life.

 In this work, a "cell-oriented" model for the structural-functional organization of the renewable zone – the shoot apical meristem (SAM) is suggested. The model is cell-oriented because its purpose is to describe the observed cell behavior (in the given case these are the types of cells and the switching over from one cell type to another) within the framework of the minimal model (Merks *et al.*, 2005). The minimal model is not intended to describe real molecular-genetic systems that control cell behavior.

The group at the growing tip of the shoot, referred to as the shoot apical meristem (SAM), is of importance. The SAM contains stem cells that continuously divide, ultimately giving rise to all the cells of the plant. Although the cells of the SAM are undifferentiated, they are determined with respect to the expression of certain genes, and, on this basis, the SAM is divided into compartments that are specifically positioned relative to each other in space through the entire life of the plant. The cells that are located within the vertical axis of the meristem in the radius of 2–4 cells at the uppermost layers 3–4 (see

Fig. 1) express, i.e. switch on-off the corresponding genes and, as a result, synthesize a protein called CLV3, belong to the central zone (CZ). Cells that express the WUS gene are located at the lower layer of the CZ cells. These cells are referred to the organizational center (OC), that is about 2–3 cells thick in the vertical direction. It is thought that the constancy of the SAM structure is required for the maintenance of the pool of the stem cells (Groß-Hardt *et al.*, 2003). The mechanisms that provide the constancy of the SAM structure are the subjects of both applied and basic in-depth studies.
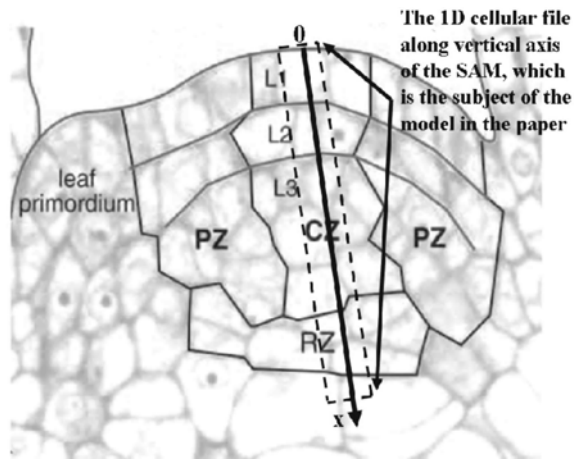

Figure 1. Cross-section of the apical meristem shoot in *Arabidopsis thaliana.*

Fig. 1 presents a cross-section of the shoot apical meristem of *Arabidopsis thaliana.* (adapted from Groß-Hardt *et al.*, 2003). The external layer is denoted by L1, the second by L2; L3 is arbitrarily called the third layer because it actually results from the cells dividing in all the planes, it is no longer a layer, rather an accumulation (a collection) of cells; CZ is the control zone, PZ is the peripheral zone, RZ is the rib-zone where cells start to differentiate into the cells of the vascular system. The X axis is pointed downwards from the shoot apex. Cells along the axis are considered as a one-dimensional array in the proposed model.

MODEL

There may be, in principle, two mechanisms that maintain the vertical compartmentalization of the meristem: first, symmetric division of the cells at the compartment boundaries with their determination in the morphogen fields and the second, asymmetric division of the cells at the boundaries. In fact, division proceeds in all the planes in the L3 layer. This makes more likely the mechanism of the vertical structure maintenance. Furthermore, in mutants whose division orientation pattern is impaired at the early stages, seedlings with a normal structural framework are formed (Berleth, Chatfield, 2002). For this reason, a possible mechanism for cell determination controlled by positional information will be considered.

Fields of the concentrations of substances that spread over from different sources (for example, by diffusion) are the physical carriers of positional information. Fig. 2 shows a possible mechanism zonal structure formation in the 1D domain. The distance is plotted along the axis X in arbitrary units which corresponds to the vertical axis that passes through the center of the apical shoot. The concentration of the morphogen Y that spreads from the apical shoot (from the point O) is plotted along the axis Y. As a result of diffusion of Y and of its continuous destruction (decay), a steady-state distribution (a

decreasing function from x) is established. At concentration above the thresholds, Y may activate gene expression in the C and W substances. It should be noted that the activation threshold for the C is higher than that for the W gene. The assumption is made that the C substance is a repressor of the expression of the W gene. It follows that where the C gene is expressed, the W gene is repressed, and the W gene is actually expressed in the zone that is remote from the shoot apex (the axis origin).
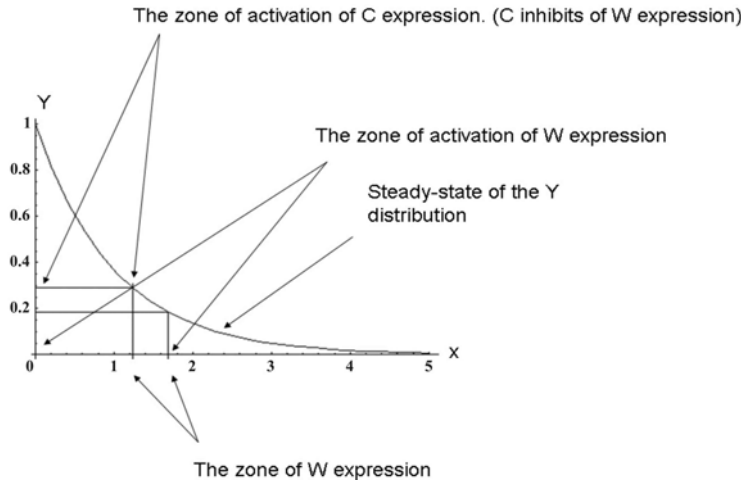

Figure 2. The simulated mechanism. The Y is distributed depending on the distance from the apical meristem and its threshold values at which the C and W expression is activated (enhanced).

$$\frac{dY(1)}{dt} = \frac{1}{\tau_Y} g(h_Y + T_{YW}W(1)) - d_Y Y(1) + D_Y (Y(2) - Y(1))$$

$$\frac{dY(i)}{dt} = -d_Y Y(i) + D_Y (Y(i-1) - 2 \cdot Y(i) + Y(i+1)) , \ 1 < i < n-1$$

$$\frac{dY(n)}{dt} = -d_Y Y(n) + D_Y (Y(n-1) - Y(n))$$

$$\frac{dC(i)}{dt} = \frac{1}{\tau_C} g(h_C + T_{CY}Y(i)) - d_C C(i) , \ \ 1 \le i \le n$$

$$\frac{dW(i)}{dt} = \frac{1}{\tau_W} g(h_W + T_{WY}Y(i) + T_{WC}C(i)) - d_W W(i) + D_W (W(i-1) - 2 \cdot W(i) + W(i+1)),$$

$$1 < i < n-1$$

$$\frac{dW(1)}{dt} = -d_W W(1) + D_W (W(2) - W(1))$$

$$\frac{dW(n)}{dt} = -d_W W(n) + D_W (W(n-1) - W(n)) ,$$

where D_S are the coefficients of S-substance diffusion, d_S are the coefficients of S-substance degradation, $\frac{1}{\tau_S}$ are the maximal rates of S substance expression. The sigmoid

function is in the form: $g(x) = \frac{1}{2}(1 + \frac{x}{\sqrt{1+x^2}})$. The arguments x of the sigmoid

functions are the liner combinations of h_S and $T_{SR} \cdot R$, where h_S – the threshold of regulation of S substance expression, and T_{SR} reflect influence of regulator R on S substance expression.

RESULTS AND DISCUSSION

The mathematical model was analyzed and its results are the subject of the paper (Nikolaev *et al.*, 2006). This simple model has numerous solutions.

An example of the model steady state solution for Y(n), W(n), and "synthesis" of W(n) is given in Fig. 3. It was shown that the proposed mechanism can stabilize the position of the OC relative to the upper point of the meristem in the vertical direction while the resident cells of the meristem compartments are substituted by the other cells. With some values of model parameters the OC positioning can periodically displace in range of 1–2 cells about a "point of attraction" (Fig. 4), which is consistent with some experimental observations.
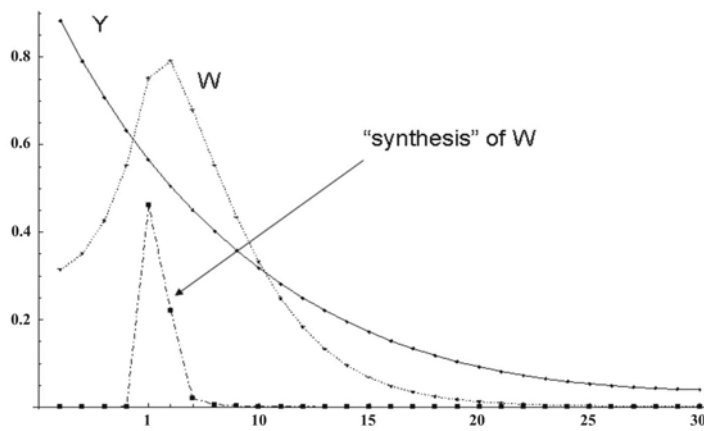

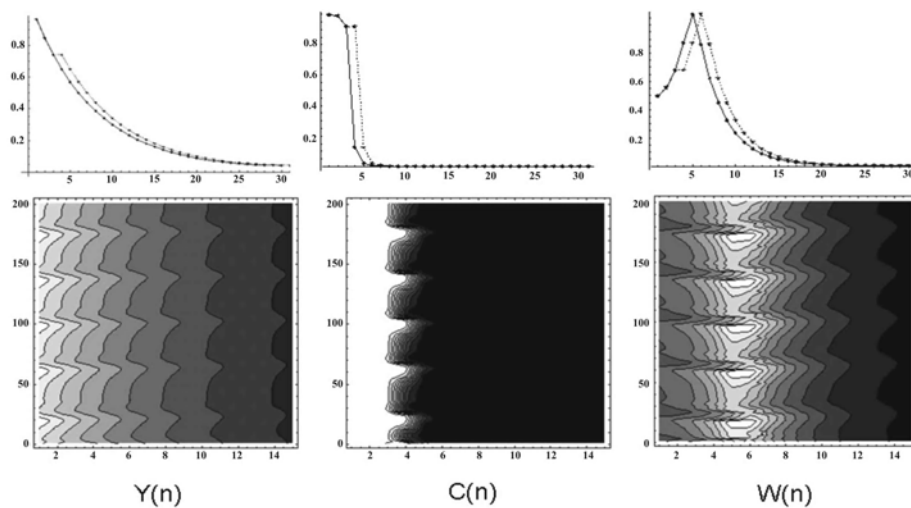*Figure 3.* A steady state solution for Y(n), W(n), and "synthesis" of W.

Figure 4. An example with 4-th cell division (upper). This perturbation of a stable solution can induce a periodical solution.

ACKNOWLEDGEMENTS

REFERENCES

Berleth Th., Chatfield S. (2002) Embryogenesis: Pattern Formation from a Single Cell. *The Arabidopsis Book. American Society of Plant Biologists.* (http://www.bioone.org/pdfserv/i1543-8120-007-01-0001.pdf).

Groß-Hardt R., Laux Th. (2003) Stem cell regulation in the shoot meristem. *J. of Cell Sci.*, **116**, 1659–1666.

Merks R.M.H., Glazier J.A. (2005) A cell-centered approach to developmental biology. *Physica A.*, **352**, 113–130.

Nikolaev S.V., Kolchanov N.A., Fadeev S.I., Kogay V.V., Mjolsness E. (2006) Analysis of a one-dimensional model for the regulation of the size of the renewable zone in biological tissue. *Computational Technologies*, **11**, 65–79 (in Russ.).